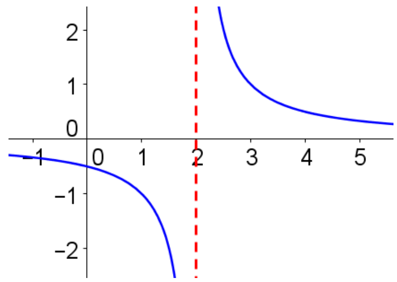
Name:___________________________

SM3: 3.3: Graphing Rational Functions

<u>Vertical Asymptote</u>: An x-value that causes division by zero because of a factor in the denominator of a rational function. Vertical Asymptotes are vertical lines that are not part of the domain of the function.

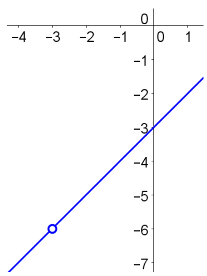
$f(x) = \frac{1}{x-2}$ will experience a vertical asymptote of $x = 2$ because the denominator contains the factor $(x - 2)$

The dashed vertical line, $x = 2$, is the vertical asymptote.

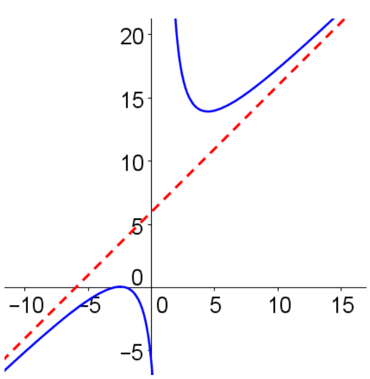
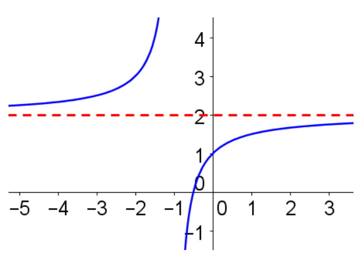
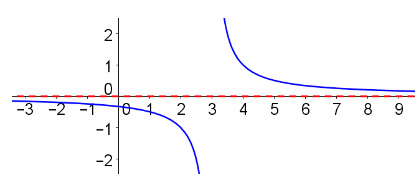


<u>Hole</u>: An x-value that causes division by zero because the same factor is in the numerator and denominator (cancellation). Holes are points that are not part of the domain of the function.

$f(x) = \frac{x^2-9}{x+3}$ can be simplified as $f(x) = x - 3; x \neq -3$. The stipulation, $x \neq -3$ means that there is a hole when x would normally be -3.



<u>End Behavior</u>: A y-value that the function approaches as the x-values get large (horizontal asymptote).

End Behavior		
Numerator has more power	Numerator ties denominator	Denominator has more power
EB: Oblique Asymptote (we will learn how to calculate and graph these soon, but not today).  As $x \to -\infty$ or $x \to \infty$, the function gets closer and closer to an oblique (slant) asymptote.	EB: $y = \frac{a}{b}$; a is the lead coefficient of the numerator and b is the lead coefficient of the denominator.  As $x \to -\infty$ or $x \to \infty$, the function gets closer and closer to $y = \frac{a}{b}$	EB: $y = 0$  As $x \to -\infty$ or $x \to \infty$, the function gets closer and closer to $y = 0$

Example: Sketch $f(x) = \frac{x+1}{x^2-2x-3}$. Describe the asymptotic and end behavior(s) using limit notation.

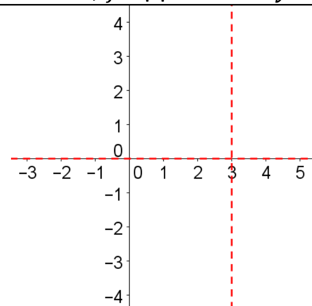
$$f(x) = \frac{x+1}{x^2-2x-3}$$

$$f(x) = \frac{x+1}{(x+1)(x-3)}$$ Factor both numerator and denominator.

$$f(x) = \frac{1}{x-3}; x \neq -1$$ Cancel like factors and list resulting stipulations.

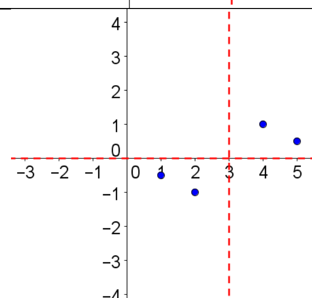
Since the denominator has a factor of $(x-3)$, f has a vertical asymptote of $x = 3$.
Since we cancelled and stipulated that $x \neq -1$, f has a hole at $x = -1$.
Since the denominator has more power than the numerator, f approaches $y = 0$ as x gets large.

Start your sketch by defining where the function can't exist by drawing your vertical asymptotes and end behaviors.

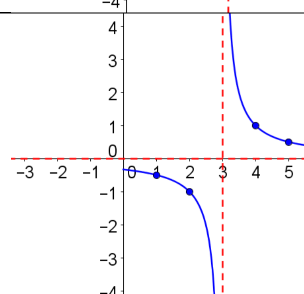


Test a couple of x-values of your function to get some idea of where your function does exist. Try a couple of points on either side of any vertical asymptotes. Make sure not to pick x-values where there are holes!

$$f(1) = -\frac{1}{2}; f(2) = -1, f(4) = 1, f(5) = \frac{1}{2}$$

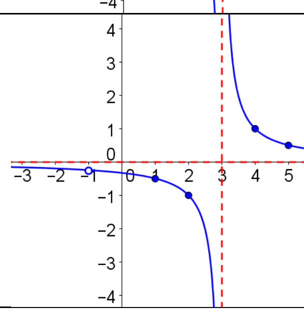


f must approach the vertical asymptote from both sides. It appears to be going down as you approach from the left and going up as you approach from the right.



As $x \to -\infty$ and $x \to \infty$, the graph must approach $y = 0$. Make sure to show the hole at $x = -1$.

Discuss the behavior near the asymptote and near the end using limit notation.



$$\lim_{x \to 3^-} f(x) = -\infty$$

$$\lim_{x \to 3^+} f(x) = \infty$$

$$\lim_{x \to -\infty} f(x) = 0$$

$$\lim_{x \to \infty} f(x) = 0$$

HW3.3

For each function, state the x-values of the vertical asymptotes (VA), holes (H), and end behaviors (EB):

1) $f(x) = \dfrac{x-2}{x+3}$

VA:

H:

EB:

2) $f(x) = \dfrac{(x-1)}{(x+5)(x-1)}$

VA:

H:

EB:

3) $f(x) = \dfrac{(x+2)^2}{x+2}$

VA:

H:

EB:

4) $f(x) = \dfrac{1}{(x-6)(x+7)}$

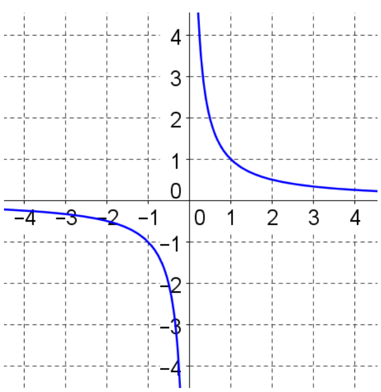
VA:

H:

EB:

Problems: Describe the asymptotic and end behavior(s) using limit notation.

5)



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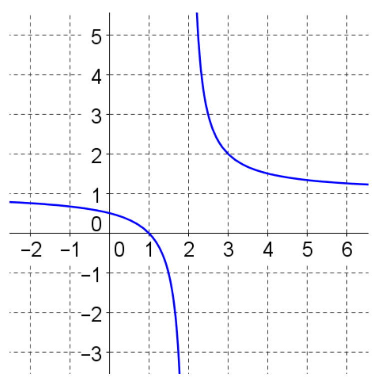

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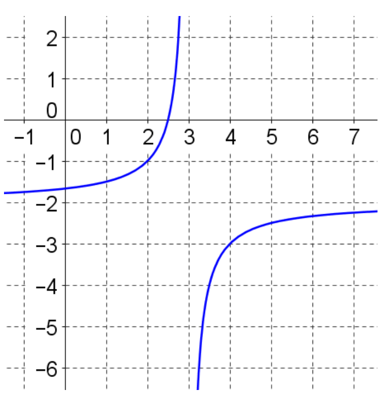

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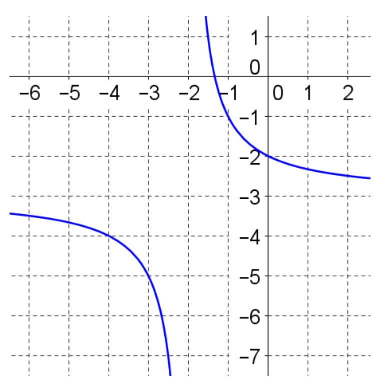

Simplify the functions (be sure to include stipulations); state the x-values of the vertical asymptotes (VA), holes (H), and end behaviors (EB):

9) $f(x) = \frac{x^2 + 2x + 1}{x^2 + 4x + 3}$

10) $f(x) = \frac{2x^2 - 5x - 12}{x^3 - 16x}$

11) $f(x) = \frac{12x^2 - 5x - 2}{9x^2 - 12x + 4}$

Simplify and sketch the function (use dashed lines for vertical asymptotes and open points for holes); describe the vertically and horizontally asymptotic behavior(s) of the function using limit notation:

12) $f(x) = \frac{x + 5}{x^2 + 7x + 10}$

13) $f(x) = \frac{x^2 - 9}{x + 3}$

14) $f(x) = \frac{x}{x^3 - 4x}$

15) $f(x) = \frac{x^2 - x - 6}{x + 2}$

16) $f(x) = \frac{1}{x^2 - 6x + 8}$

17) $f(x) = \frac{-(x - 3)}{x^2 - 4x + 3}$

18) $f(x) = -\frac{1}{x^2 - 4x}$

19) $f(x) = \frac{x^2 - 2x - 8}{x + 2}$

20) $f(x) = \frac{5x^2 - 5}{x^2 - 1}$

Problem Creation: Graph functions that exhibits the following properties:

21) Sketch $f(x)$, as described below:

$\lim_{x \to 1^-} f(x) = \infty$

$\lim_{x \to 1^+} f(x) = -\infty$

$f(x)$ is strictly increasing

$D_f = (-\infty, 1) \cup (1,3) \cup (3, \infty)$

22) Sketch $g(x)$, as described below:

$\lim_{x \to -2^-} g(x) = \infty$; $\lim_{x \to 2^-} g(x) = -\infty$

$\lim_{x \to -2^+} g(x) = -\infty$; $\lim_{x \to 2^+} g(x) = \infty$

$g(x)$ has even symmetry

$D_g = (-\infty, -2) \cup (-2,0) \cup (0,2) \cup (2, \infty)$